Background (H.1)

전달함수 $H(s) = C\,\underbrace{(sI - A)^{-1}}_{\Phi(s)}\,B$

$$\Updownarrow \qquad\qquad \Updownarrow$$

임펄스응답 $h(t) = C\,\underbrace{e^{At}}_{\Phi(t)}\,B$

$\Phi(t)$ ~~~ 상태천이행렬

State transition Matrix

$$\frac{1}{s+a}$$

$$e^{-at}$$

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

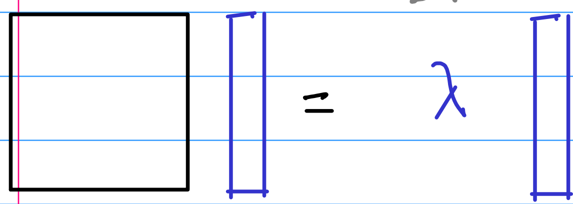
$$At = t\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_1 t & a_2 t & a_3 t \\ b_1 t & b_2 t & b_3 t \\ c_1 t & c_2 t & c_3 t \end{bmatrix}$$

$$e^{At} = \cancel{\begin{bmatrix} e^{a_1 t} & e^{a_2 t} & e^{a_3 t} \\ e^{b_1 t} & e^{b_2 t} & e^{b_3 t} \\ e^{c_1 t} & e^{c_2 t} & e^{c_3 t} \end{bmatrix}}$$

a given matrix A

$$Ap = \lambda p$$

스라



λ ?

$$-Ap + \lambda p = 0$$

$$\lambda I p - Ap = 0$$

$$(\lambda I - A)p = 0$$

$$\phi(s) = (sI - A)^{-1}$$

$$[A]\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(\lambda \mathbf{I} - \mathbf{A})\mathbf{p} = \mathbf{0}$$

① if $(\quad)^{-1}$ exists

$$\mathbf{p} = \underset{n\times n}{(\lambda \mathbf{I} - \mathbf{A})^{-1}} \underset{n\times 1}{\mathbf{0}} = \mathbf{0}$$

$\mathbf{p} = \mathbf{0}$ the unique solution

② if $(\quad)^{-1}$ does not exist $\Rightarrow$ $|\lambda \mathbf{I} - \mathbf{A}| = 0$

there exists non-zero $\mathbf{p} \neq \mathbf{0}$

such that $(\lambda \mathbf{I} - \mathbf{A})\mathbf{p} = \mathbf{0}$

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda+2 & -1 \\ -1 & \lambda+2 \end{bmatrix}$$

$$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$\lambda = -1$ $\qquad$ $\lambda = -3$

$$|\lambda I - A| = 0 \Longrightarrow (\lambda+2)^2 - 1 = 0$$

$$\lambda^2 + 4\lambda + 3 = 0$$

$$(\lambda+3)(\lambda+1) = 0$$

$$\lambda = -1, \ -3$$

A's eigenvalues

$$f(x) = x^2 + 4x + 3 = 0 \Rightarrow x = -1, \quad x = -3$$

A의 eigen value를 구하기 위한 식.

$$f(A) = A^2 + 4A + 3I = 0$$

Caley-Hamilton Theorem

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \qquad (\lambda I - A) = \begin{bmatrix} \lambda+2 & -1 \\ -1 & \lambda+2 \end{bmatrix}$$

$$(\lambda I - A) = \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$\lambda = -1$

$$(\lambda I - A) = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$\lambda = -3$

$\boxed{\lambda = -1}$ 에 대한 eigenvector를 구하자

$$(\lambda I - A) \cdot p = 0$$

$$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{aligned} a - b &= 0 \\ -a + b &= 0. \end{aligned}$$

$\lambda = -1$

$$a = b = \underline{1}$$

$$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = -1$ 일 때 eigenvector

$\boxed{\lambda = -3}$ 에 대한 eigenvector를 구하자

$$(\lambda I - A) \cdot p = 0$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{aligned} -a - b &= 0 \\ -a - b &= 0. \end{aligned}$$

$\lambda = -3$

$$a = -b = \underline{1}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = -3$

$\lambda = -3$ 일 때 eigenvector

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

eigenvalu eigenvector

$\lambda = -1$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

eigenvalu eigenvetor

$\lambda = -3$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \overset{-1}{=} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \overset{-3}{=} \begin{bmatrix} -3 \\ +3 \end{bmatrix}$$

$$A p = (-1)\, p \qquad A p = (-3)\, p$$

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$|\lambda I - A| = 0$$

to find eigen value

$$\begin{vmatrix} \lambda+2 & -1 \\ -1 & \lambda+2 \end{vmatrix} = (\lambda+2)^2 - 1$$

$$= \lambda^2 + 4\lambda + 3$$

$$= (\lambda+1)(\lambda+3)$$

$\lambda_1 = -1$, $\lambda_2 = -3$

$$f(x) = x^2 + 4x + 3 = 0 \qquad f(-1) = 0 \quad f(-3) = 0$$

$$f(A) = A^2 + 4 \cdot A + 3I = \mathbf{0}$$

$$A^2 = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$f(A) = A^2 + 4 \cdot A + 3I \Rightarrow \mathbf{0}$$

$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} + 4\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} + 3I$$

Caley-Hamilton Theorem

$$= \begin{bmatrix} 5-8+3 & -4+4+0 \\ -4+4+0 & 5-8+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Diagonalization

$$\underset{n \times n}{A}$$

$\lambda_1, \lambda_2, \cdots, \lambda_n$; n개의 distinct eigenvalue

$\downarrow \quad \downarrow \quad \quad \downarrow$

$p_1 \quad p_2 \quad \cdots \quad p_n$ eigenvector.

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix} = \underset{n \times n}{P}$$

$$A \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix} = A P$$

$$\begin{bmatrix} A p_1 & A p_2 & \cdots & A p_n \end{bmatrix}$$

$A p_1 = \lambda_1 p_1$

$A p_2 = \lambda_2 p_2$

$A p_n = \lambda_n p_n$

$$P \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{bmatrix} \rightarrow \Lambda$$

$$AP = P\Lambda$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1x_1 + a_2x_2 + a_3x_3 & a_1y_1 + a_2y_2 + a_3y_3 & a_1z_1 + a_2z_2 + a_3z_3 \\ b_1x_1 + b_2x_2 + b_3x_3 & b_1y_1 + b_2y_2 + b_3y_3 & b_1z_1 + b_2z_2 + b_3z_3 \\ c_1x_1 + c_2x_2 + c_3x_3 & c_1y_1 + c_2y_2 + c_3y_3 & c_1z_1 + c_2z_2 + c_3z_3 \end{bmatrix}$$

$$= \left[\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \right]$$

$$\mathbf{A}\mathbf{P}_1 \qquad \mathbf{A}\mathbf{P}_2 \qquad \cdots \qquad \mathbf{A}\mathbf{P}_n$$

$$= \lambda_1 \mathbf{P}_1 \qquad = \lambda_2 \mathbf{P}_2 \qquad \cdots \qquad = \lambda_n \mathbf{P}_n$$

$$= \left[\lambda_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \lambda_2 \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \lambda_n \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \right]$$

$$= \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 x_1 & a_1 y_1 & a_1 z_1 \\ b_2 x_2 & b_2 y_2 & b_2 z_2 \\ c_3 x_3 & c_3 y_3 & c_3 z_3 \end{bmatrix}$$

$\star$

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 x_1 & b_2 y_1 & c_3 z_1 \\ a_1 \cdot x_2 & b_2 y_2 & c_3 z_2 \\ a_1 x_3 & b_2 y_3 & c_3 z_3 \end{bmatrix}$$

$$= \left[a_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b_2 \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad c_3 \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \right]$$

$$\lambda_1 \mathbf{p}_1 \qquad \lambda_2 \mathbf{p}_2 \qquad \lambda_n \mathbf{p}_n$$

$$AP = P\Lambda$$

$$P^{-1}AP = \Lambda$$

$$A \rightarrow P \; \cdots \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}$$

eigenvectors

$$P^{-1} A P = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

eigenvalues

$$AP = P\Lambda$$

$$P^{-1}AP = \Lambda$$

$$A = P\Lambda P^{-1}$$

$$A^2 = AA = (P\Lambda \cancel{P^{-1}})(\cancel{P}\Lambda P^{-1}) = P\Lambda^2 P^{-1}$$

$$A^k = P\Lambda^k P^{-1}$$

$$= P \boxed{\begin{matrix} \lambda_1^k & & \\ & \lambda_2^k & \\ & & \lambda_n^k \end{matrix}} P^{-1}$$

$$\Lambda^2 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1^2 & 0 & 0 \\ 0 & b_2^2 & 0 \\ 0 & 0 & c_3^2 \end{bmatrix}$$

$$\Lambda^k = \begin{bmatrix} a_1^k & 0 & 0 \\ 0 & b_2^k & 0 \\ 0 & 0 & c_3^k \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}^k$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} & & \\ & \Lambda^{-1}\,? & \\ & & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} a_1^{-1} & 0 & 0 \\ 0 & b_2^{-1} & 0 \\ 0 & 0 & c_3^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_n \end{bmatrix} \qquad \Lambda t = \begin{bmatrix} \lambda_1 t & & \\ & \lambda_2 t & \\ & & \lambda_n t \end{bmatrix}$$

$$e^{\Lambda t} \equiv \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & e^{\lambda_n t} \end{bmatrix}$$

Taylor Series

$$f(x) = e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{\lambda t} = 1 + \frac{\lambda t}{1!} + \frac{\lambda^2 t^2}{2!} + \frac{\lambda^3 t^3}{3!} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda^k$$

$$e^A = ?$$

$$f(x) = e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$f(A) = e^A = 1 + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$f(t) = e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$$

$$f(A) = e^A \triangleq 1 + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

* Use Taylor Series.

$$e^A = e^{\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}} \qquad \cancel{\begin{bmatrix} e^2 & 0 \\ e^0 & e^0 \end{bmatrix}}$$

$$= I + \frac{1}{1!}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^2 + \frac{1}{3!}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^3 + \cdots$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^3 = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= I + \frac{1}{1!}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} 2^2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{3!}\begin{bmatrix} 2^3 & 0 \\ 0 & 0 \end{bmatrix} + \cdots$$

$$= \begin{bmatrix} 1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^2 & 0 \\ 0 & e^0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

* Use Taylor Series.

$$e^A = e^{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}} \cdot \cancel{\begin{bmatrix} e^2 & e^0 \\ e^0 & e^3 \end{bmatrix}}$$

$$= I + \frac{1}{1!}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^2 + \frac{1}{3!}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^3 + \cdots$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^2 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^3 = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 27 \end{bmatrix}$$

$$= I + \frac{1}{1!}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} 2^2 & 0 \\ 0 & 3^2 \end{bmatrix} + \frac{1}{3!}\begin{bmatrix} 2^3 & 0 \\ 0 & 3^3 \end{bmatrix} + \cdots$$

$$= \begin{bmatrix} 1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots & 0 \\ 0 & 1 + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \cdots \end{bmatrix} = \begin{bmatrix} e^2 & 0 \\ 0 & e^3 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$e^{\Lambda t} = e^{\begin{bmatrix} \lambda_1 t & & 0 \\ & \lambda_2 t & \\ 0 & & \ddots \quad \lambda_n t \end{bmatrix}}$$

$$= I + \frac{1}{1!}\begin{bmatrix} & \end{bmatrix}^1 + \frac{1}{2!}\begin{bmatrix} & \end{bmatrix}^2 + \frac{1}{3!}\begin{bmatrix} & \end{bmatrix}^3 + \cdots$$

$$\begin{bmatrix} \diagdown \end{bmatrix}^1 = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & \ddots & \lambda_n \end{bmatrix}$$

$$\begin{bmatrix} \diagdown \end{bmatrix}^2 = \begin{bmatrix} \lambda_1^2 & & 0 \\ & \lambda_2^2 & \\ 0 & \ddots & \lambda_n^2 \end{bmatrix}$$

$$\begin{bmatrix} \diagdown \end{bmatrix}^3 = \begin{bmatrix} \lambda_1^3 & & 0 \\ & \lambda_2^3 & \\ 0 & \ddots & \lambda_n^3 \end{bmatrix}$$

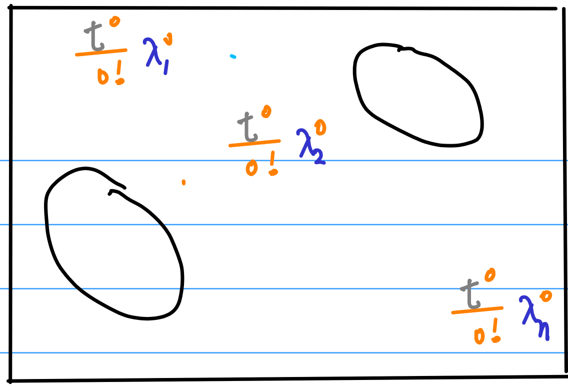
$$e^{\Lambda t} = \begin{bmatrix} \sum_{k=0}^{\infty} \frac{\lambda_1^k}{k!} t^k & & & 0 \\ & \sum_{k=0}^{\infty} \frac{\lambda_2^k}{k!} t^k & & \\ 0 & & \ddots & \\ & & & \sum_{k=0}^{\infty} \frac{\lambda_n^k}{k!} t^k \end{bmatrix}$$

$$e^{\lambda t} = \boxed{\sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda^k}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & e^{\lambda_n t} \end{bmatrix}$$

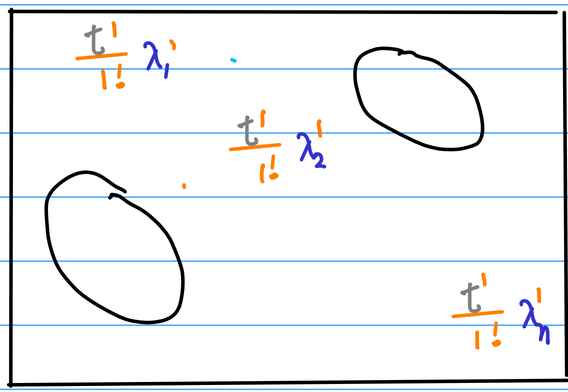
$$e^{\Lambda t} = \begin{bmatrix} \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_1^k & & \\ & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_2^k & \\ & & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_n^k \end{bmatrix}$$

$k=0$



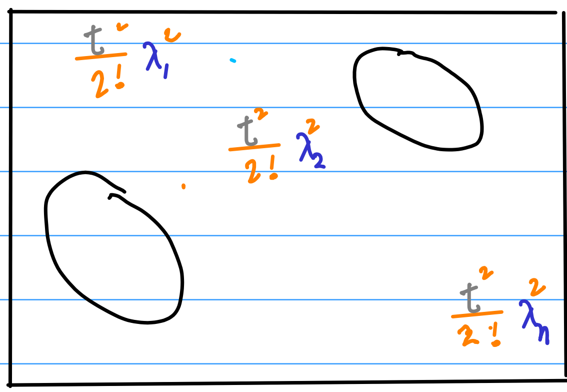
$= \frac{t^0}{0!}\Lambda^0$

$k=1$



$= \frac{t^1}{1!}\Lambda^1$

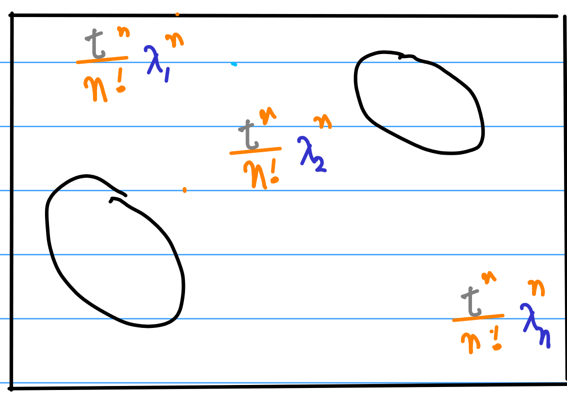
$k=2$



$= \frac{t^2}{2!}\Lambda^2$

$\vdots$

$k=n$



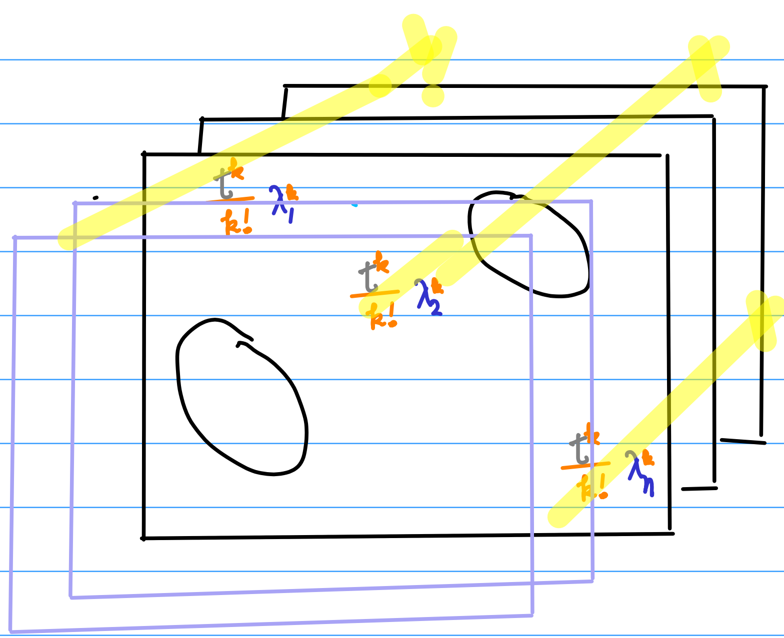
$= \frac{t^n}{n!}\Lambda^n$

$\vdots$

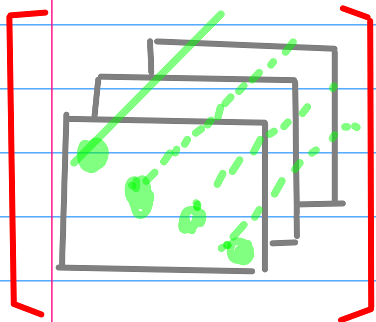
$$e^{\Lambda t} = \sum_{k=0}^{\infty} \frac{t^k}{k!}\Lambda^k$$

$$e^{\Lambda t} = \sum_{k=0}^{\infty} \frac{t^k}{k!} \Lambda^k$$

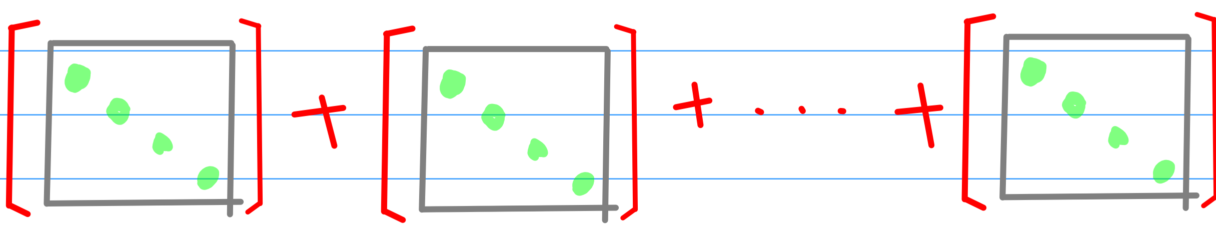
$$= \begin{bmatrix} \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_1^k & & \\ & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_2^k & \\ & & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_n^k \end{bmatrix}$$



$$e^{\Lambda t} = \begin{bmatrix} \sum_{k=0}^{\infty} \frac{\lambda_1^k}{k!} t^k & & & \\ & \sum_{k=0}^{\infty} \frac{\lambda_2^k}{k!} t^k & & \\ & & \ddots & \\ & & & \sum_{k=0}^{\infty} \frac{\lambda_n^k}{k!} t^k \end{bmatrix}.$$



$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} \begin{bmatrix} \lambda_1^k & & & \\ & \lambda_2^k & & \\ & & \ddots & \\ & & & \lambda_n^k \end{bmatrix}.$$



$$\boxed{e^{\Lambda t} = \sum_{k=0}^{\infty} \frac{t^k}{k!} \Lambda^k}$$

Taylor Series

$$\boxed{f(t) = e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots}$$

$$f(t) = e^t = \sum_{k=0}^{\infty} \frac{t^k}{k!}$$

$$\boxed{f(A) = e^A = 1 + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots}$$

$$f(A) = e^A = \sum_{k=0}^{\infty} \frac{A^k}{k!}$$

$$e^{\Lambda} = \sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} \begin{bmatrix} \lambda_1^k & & 0 \\ & \lambda_2^k & \\ 0 & & \ddots \; \lambda_n^k \end{bmatrix}$$

$$\boxed{e^{\Lambda} = \sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}}$$

$$Pe^{\Lambda}P^{-1} = P\left(\sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}\right)P^{-1}$$

$$= \sum_{k=0}^{\infty} \frac{P\Lambda^k P^{-1}}{k!} \Longrightarrow A^k$$

$$\boxed{AP = P\Lambda \quad \begin{cases} A = P\Lambda P^{-1} \\ \Lambda = P^{-1}AP \end{cases}}$$

$$P\Lambda^k P^{-1} = P(P^{-1}AP)^k P^{-1}$$

$$= P\overbrace{(P^{-1}AP)(P^{-1}AP)\cdots(P^{-1}AP)}^{k\text{번}}P^{-1}$$

$$= A^k$$

$$P e^{\Lambda t} P^{-1} = P\left(\sum_{k=0}^{\infty} \frac{t^k}{k!} \Lambda^k\right) P^{-1}$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} \boxed{P \Lambda^k P^{-1}}$$

$$\overbrace{\underbrace{\boxed{P \Lambda P^{-1}}}_{A}\ \underbrace{\boxed{P \Lambda P^{-1}}}_{A} \cdots \underbrace{\boxed{P \Lambda P^{-1}}}_{A}}^{k} = A^k$$

$$\boxed{e^{\Lambda} = \sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}}$$

$$P e^{\Lambda} P^{-1} = \sum_{k=0}^{\infty} \frac{P \Lambda^k P^{-1}}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{A^k}{k!} = e^{A}$$

$$\boxed{P \Lambda^k P^{-1} = A^k}$$

$$\boxed{e^{A} = \sum_{k=0}^{\infty} \frac{A^k}{k!}} \qquad \boxed{e^{A} = P e^{\Lambda} P^{-1}}$$

$$\boxed{e^{\Lambda} = \sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}} \qquad \boxed{e^{\Lambda} = P^{-1} e^{A} P}$$

$$A = P \Lambda P^{-1}$$

$$\Lambda = P^{-1} A P$$

$$e^{at} = \sum_{k=0}^{\infty} \frac{t^k}{k!} a^k \qquad e^{At} = \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k$$

$$\boxed{\begin{aligned} e^{a(t_1+t_2)} &= e^{at_1} \cdot e^{at_2} \\ e^{at} \cdot e^{-at} &= 1 \\ (e^{at})^{-1} &= e^{-at} \\ \frac{d}{dt}(e^{at}) &= a e^{at} \end{aligned}} \qquad \boxed{\begin{aligned} e^{A(t_1+t_2)} &= e^{At_1} \cdot e^{At_2} \\ e^{At} \cdot e^{-At} &= I \\ (e^{At})^{-1} &= e^{-At} \\ \frac{d}{dt}(e^{At}) &= A e^{At} \end{aligned}}$$

$$\boxed{AP = P\Lambda \quad \therefore \begin{cases} A = P\Lambda P^{-1} \\ \Lambda = P^{-1}AP \end{cases}}$$

$$\begin{array}{cc} & n \\ n & \boxed{A} \end{array}$$

$\lambda_1, \lambda_2, \cdots, \lambda_n$; n개의 distinct eigenvalue

$$\lambda_1 \downarrow \quad \lambda_2 \downarrow \quad \lambda_n \downarrow$$

$$\boxed{p_1} \quad \boxed{p_2} \quad \boxed{p_n}$$

eigenvector.

$$n \begin{bmatrix} p_1 & p_2 & p_n \end{bmatrix} = n \overset{n}{\boxed{P}} \qquad n \overset{n}{\begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_n \end{bmatrix}} \Lambda$$

$\Lambda \leftarrow A$ $\qquad$ $A \leftarrow \Lambda$

$\Lambda = P^{-1} A P$ $\qquad$ $A = P \Lambda P^{-1}$

$\Lambda^k = P^{-1} A^k P$ $\qquad$ $A^k = P \Lambda^k P^{-1}$

$e^{\Lambda} = \sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}$ $\qquad$ $e^{A} = \sum_{k=0}^{\infty} \frac{A^k}{k!}$

$e^{\Lambda} \doteq P^{-1} e^{A} P$ $\qquad$ $e^{A} \doteq P e^{\Lambda} P^{-1}$

$$\Lambda \leftarrow A \qquad\qquad A \leftarrow \Lambda$$

$$(sI-\Lambda) = P^{-1}(sI-A)P \qquad (sI-A) = P(sI-\Lambda)P^{-1}$$

$$(sI-\Lambda)^{-1} = P^{-1}(sI-A)^{-1}P \qquad (sI-A)^{-1} = P(sI-\Lambda)^{-1}P^{-1}$$

$$e^{\Lambda t} = P^{-1}e^{At}P \qquad e^{At} = Pe^{\Lambda t}P^{-1} = \varphi(t)$$

$$e^{\Lambda} = \sum_{k=0}^{\infty}\frac{\Lambda^k}{k!} \qquad e^{A} = \sum_{k=0}^{\infty}\frac{A^k}{k!}$$

$$e^{\Lambda} \doteq P^{-1}e^{A}P \qquad e^{A} \doteq Pe^{\Lambda}P^{-1}$$